Large-Signal Stability Analysis of Pulsed Constant Power Loads via Sum-of-Squares Optimization

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***Abstract*— Future sensors and weapons rapidly vary their power consumption presenting new challenges to ensuring stable operation of naval power systems. Through a series of examples, it is shown that standard small-signal impedance methods are a poor surrogate for understanding these pulsed systems and there is a need for large-signal stability analysis tools. Sum-of-squares optimization is proposed as a means of analyzing the nonlinear dynamics of these systems by providing estimates of the region of attraction, a key metric in large-signal stability. It is compared to polytopic methods and seen to provide more accurate results.**

I. INTRODUCTION

Naval power systems are increasingly moving towards power electronic distribution systems (PEDS). Such systems are advantageous for their high power density, efficiency, and configurability. High-bandwidth power electronics ensure tight output regulation, rejecting input voltage disturbances from propagating to the output. As a result of their controls, power electronic converters often behave as constant power loads which, in a small-signal sense, exhibit a negative incremental impedance. This negative impedance may destabilize the input filter of the converter if the filter is not properly damped [1]. Similarly, even if the filter and converter are stable when supplied by an ideal (zero impedance) source, instability may emerge when connected to a power system with non-negligible source impedance.

Both of these problems, internal stability and interconnected system stability respectively, have been extensively studied. The prevailing method for DC systems is to assess stability by examining the impedance ratio of the source impedance to load impedance at a given interface [2]. This ratio is plotted on the Nyquist plot. Given an independently stable source and load, encirclement of the $(-1, 0j)$ point indicates the interconnected system will be unstable. If the Nyquist plot does not encircle this point, the system will be stable in a small-signal sense. Similar impedance-based methods exist for AC systems through the use of rotating reference frames which transform the sinusoidal signals to DC signals, allowing the system equations to be linearized [3].

Both AC and DC impedance methods are a form of small-signal (linear) analysis. Small-signal analysis gives no indication of the magnitude of perturbations from which the system can stably recover. Further, it provides no proof that the system can successfully transition between different small-signal stable operating points. Addressing these concerns is the focus of a large-signal stability analysis. For a review of definitions, the reader is referred to [4].

PEDs admit a near-infinite number of possible operating points due to changes in both load power consumption (e.g. a motor drive draws more power at full speed, maximum torque) and plant configuration (sources and loads being connected/disconnected). While it is impossible to examine every possible operating point, methods exist [5] for conservatively bounding the set of source impedances and load impedances to ensure that all operating points are small-signal stable. It is standard practice to require that all operating points have a minimum gain margin (typically 3dB to 6dB) and phase margin (typically $30° - 60°$) on the Nyquist plot. A review of small-signal stability criteria for DC systems can be found in [2].

While the impedance methods give no indication of large-signal stability, for systems that slowly vary their operating point (e.g. a motor drive ramping up), having all operating points be small-signal stable typically results in a system that can stably transition between operating points. Multiple aircraft and naval power systems ships have been successfully integrated while relying only on small-signal analyses plus simulation to ensure system stability.

Future sensors and weapons such as high-power radars, railguns, and high-energy lasers present new challenges to the existing methods of stability analysis due to their rapid variations in power consumption (and associated operating point). This can result in significant voltage swings on the input supply voltage, causing large deviations from the equilibrium condition at which point the linear analysis is no longer valid. This paper demonstrates how nonlinear analysis tools relying on sum-of-squares optimization may be utilized to analyze large-signal stability in pulsed constant power loads. By pulsed constant power load we mean a load that instantaneously switches between different power consumption levels (e.g. off/on) and draws current to exactly consume the present power setting.

A. Related Work and Contributions

To the author's knowledge, there is little work addressing large-signal stability of pulsed loads from an analytical (vice simulation-based) standpoint. In [6] the author's study the stability of periodic pulsed loads using an extension of Lyapunov techniques known as a Hamiltonian surface. However, the method ultimately relies on extensive numerical simulation due to the lack of a closed form representation of the model.

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We leverage two well-established techniques (polytopic modeling, sum-of-squares (SOS) optimization) for obtaining an estimate of a nonlinear system's region of attraction (ROA). Both methods have been successfully applied to analyzing constant power loads in existing works [7], [8], [9]. Our main contribution is to show how these techniques can be leveraged for analyzing pulsed loads. Additionally, we provide a comparison on realistic models which reveals the strength of sum-of-squares optimization. Finally, our examples demonstrate how small-signal metrics (gain margin, phase margin) are a poor surrogate for large-signal stability of pulsed loads.

B. Organization

The rest of this paper is organized as follows. Section II provides a brief review of local stability analysis and methods for obtaining an estimate of the region of attraction. Section III studies the large-signal stability of three pulsed load models of increasing complexity. Section IV concludes the paper.

II. PRELIMINARIES

We first briefly review sum-of-squares optimization. This material is largely summarized from [10], [11] to which the reader is referred for a more comprehensive introduction. We then review how these methods can be applied to Lyapunov-based stability analysis.

A. Sum-of-Squares Optimization

Let $x_1, \ldots, x_n$ denote the elements of a vector variable $x \in \mathbb{R}^n$. Recall that a monomial is a product of variables $x_1, \ldots, x_n$ with non-negative integer exponents. The degree d of a monomial is the sum of the exponents. For example, $x_1^3 x_2$ is a monomial of degree 4. A polynomial is a finite linear combination of monomials. The degree of a polynomial is the highest degree of its monomials.

Definition 1 ([10])**.** A polynomial $p(x)$ is a sum-of-squares polynomial if there exists polynomials $g_1(x), \ldots, g_k(x)$ such that $p(x) = \sum_{i=1}^{k} g_i(x)^2$.

Remark. It is obvious that any SOS polynomial is non-negative for all $x \in \mathbb{R}^n$.

Let $\sum[x]$ denote the set of SOS polynomials in the vector variable x. We indicate that a given polynomial is an SOS polynomial by $p(x) \in \sum[x]$. The following theorem establishes a link between SOS polynomials and positive semidefinite matrices.

Theorem 1 ([10])**.** *A polynomial $p(x)$ of degree $2d$ is a sum-of-squares polynomial if and only if there exists a positive semidefinite matrix Q such that $p(x) = z(x)^T Q z(x)$ where $z(x)$ is a vector of monomials up to degree d.*

The form $z(x)^T Q z(x)$ is called a Gram matrix representation. If the Gram matrix Q is positive semidefinite than $z(x)^T Q z(x) \geq 0 \; \forall \; x \in \mathbb{R}^n$ and we can write the polynomial $p(x)$ as a sum-of-squares. The following example taken from [11] demonstrates this decomposition.

$$\begin{aligned}
p(x) &= x_1^2 + 2x_1^4 + 2x_1^3 x_2 - x_1^2 x_2^2 + 5x_2^4 \\
&= \begin{bmatrix} x_1 \\ x_1^2 \\ x_1 x_2 \\ x_2^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & -0.5 \\ 0 & 1 & 0 & 0 \\ 0 & -0.5 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1^2 \\ x_1 x_2 \\ x_2^2 \end{bmatrix} \\
&= x_1^2 + \frac{1}{2}(2x_1^2 - 3x_2^2 + x_1 x_2)^2 + \frac{1}{2}(x_2^2 + 3x_1 x_2)^2
\end{aligned}$$

Given a specified vector of monomials $z(x)$, the search for a matrix Q yielding an SOS decomposition $p(x) = z(x)^T Q z(x)$ can be formulated as a linear matrix inequality which can be readily solved using semidefinite progamming.

B. Lyapunov Stability Analysis

Consider an autonomous nonlinear dynamical system of the form:

$$\dot{x} = f(x(t)) \tag{1}$$

Where $x \in \mathbb{R}^n, f : \mathbb{R}^n \to \mathbb{R}^n$ and f is locally Lipschitz on $\mathbb{R}^n$. By an appropriate change of coordinates, let the origin be an equilibrium point, $f(0) = 0$. Denote by $\phi(x_0, t)$ the solution to (1) with initial condition $x(0) = x_0$.

Definition 2 ([13])**.** The region of attraction is defined as the set of all initial conditions that converge back to the equilibrium point: $\Omega = \{x \in \mathbb{R}^n : \lim_{t \to \infty} \phi(x_0, t) = 0\}$.

Determining the exact ROA is difficult for general nonlinear systems. Instead, most methods focus on obtaining inner estimates of the ROA by finding invariant subsets which are defined as follows.

Definition 3 ([13])**.** A set of states $\mathcal{M}$ is called an invariant set of (1) if for all $x_0 \in \mathcal{M}, \phi(x_0, t) \in \mathcal{M} \; \forall \; t \geq 0$.

A Lyapunov function $V(x)$ characterizes invariant subsets of the ROA:

Theorem 2 ([13])**.** *Let γ be a positive scalar. If there exists a continuously differentiable function $V(x) : \mathbb{R}^n \to \mathbb{R}$ such that:*

$$\begin{aligned}
V(0) = 0, V(x) > 0 \; \forall \; x \neq 0 \\
\Omega(V, \gamma) = \{x \in \mathbb{R}^n : V(x) \leq \gamma\} \textit{is bounded} \\
\Omega(V, \gamma) \subset \{x \in \mathbb{R}^n : \nabla V(x) f(x) < 0\}
\end{aligned}$$

Then for all $x \in \Omega(V, \gamma)$ the solution $\phi(x, t) \in \Omega(V, \gamma) \; \forall \; t \geq 0$ and $\lim_{t \to \infty} \phi(x_0, t) = 0$

C. Region of Attraction Estimates via Sum-of-Squares

In general, finding Lyapunov functions to prove global or local stability of dynamic systems is a difficult process when done manually. By restricting our search to polynomial dynamic systems and polynomial Lyapunov functions, we can leverage sum-of-squares optimization to find candidate Lyapunov functions satisfying the conditions of Theorem 2.

For example, proving global stability can be done with a Lyapunov function satisfying the following conditions:

$$V(x) > 0 \;\forall\; x \neq 0, V(0) = 0$$
$$\nabla V(x) f(x) < 0 \;\forall\; x \neq 0$$

If we limit our search to SOS polynomial Lyapunov functions, these conditions can be formulated as the following SOS optimization problem:

$$\begin{aligned} V(x) - l_1(x) &\in \sum[x] \\ -(\nabla V(x) f(x) - l_2(x)) &\in \sum[x] \\ V(0) &= 0 \end{aligned}$$

Recall that SOS polynomials are non-negative while Lyapunov functions must satisfy strictly positive (or strictly negative) conditions. The functions $l_1(x)$ and $l_2(x)$ are positive definite functions that ensure the resulting inequalities are strict (i.e. $V(x) > 0$ for $x \neq 0$). A common choice is $l(x) = \epsilon x^T x$ where ϵ is 10^{-6}.

For systems that are not globally stable, we instead aim to prove local stability by finding an estimate of the ROA. The previous problem can be modified using a variant of the S-procedure [14].

$$\begin{aligned} \max \quad & \gamma \\ \text{s.t.} \quad & V(x) - l_1(x) \in \sum[x], \\ & -(\nabla V(x) f(x) - l_2(x)) - s_1(x)(V(x) - \gamma) \in \sum[x], \\ & s_1(x) \in \sum[x] \end{aligned}$$

This optimization problem attempts to find the largest set $\Omega(V, \gamma)$ of a given Lyapunov function $V(x)$ that ensures stability. This set is then an estimate of the ROA. The term $s_1(x)(V(x) - \gamma)$ arises from applying the S-procedure. It serves to relax the condition $\nabla V(x) f(x) < 0$ to only be required for $x \in \Omega(V, \gamma)$.

A given locally stable system may admit multiple Lyapunov functions, each of which provides a different inner estimate of the ROA. By introducing a shape function $h(x)$ it is possible to find Lyapunov functions that maximize the inner estimate in a given direction. This is done by solving the following SOS optimization problem.

$$\begin{aligned} \max \quad & \beta \\ \text{s.t.} \quad & V(x) - l_1(x) \in \sum[x], \\ & -(\nabla V(x) f(x) - l_2(x)) - s_1(x)(V(x) - \gamma) \in \sum[x], \\ & -(\beta - h(x)) s_2(x) + (V(x) - \gamma) \in \sum[x], \\ & s_1(x), s_2(x) \in \sum[x] \end{aligned}$$

The decision variables are $V(x), s_1(x), s_2(x), \gamma$, and β. The resulting optimization problem contains products of decision variables $(s_1(x)V(x), \beta s_2(x)$ which leads to a bilinear semidefinite program which is nonconvex and difficult to solve. A common alternative is the V-s iteration in which one alternates between optimizing one term of a bilinear expression which holding the other fixed [11]. This method was used for the results that follow.

D. Region of Attraction Estimates via Polytopic Modeling

Sum-of-squares works directly with nonlinear polynomial dynamics. Alternatively, polytopic (or Takagi-Sugeno) modeling may be used to represent a nonlinear model through a convex combination of linear models. Given q nonlinearities in the model each of which has a minimum and maximum value, the nonlinear model can be represented by a weighted combination of 2^q linear models, each with state matrix $A_i, i = 1, \ldots, 2^q$. These represent the vertices of the polytope formed from all combinations of the minimum and maximum values of the nonlinearities. Stability can then be established by finding a common quadratic Lyapunov function $V(x) = x^T P x$ for the family of matrices A_i where P is positive definite [14]. If the polytopic model is a valid representation of the nonlinear model over the whole state space $\mathbb{R}^n$, global stability follows. If the polytopic model is valid only over a subspace $\mathcal{S} \subset \mathbb{R}^n$ then an estimate of the region of attraction is given by $\Omega(V, \gamma) = \{x \in \mathbb{R}^n : V(x) \leq \gamma\}$ where γ is the maximum level set of $V(x)$ contained within $\mathcal{S}$. The resulting linear matrix inequality is given by:

$$\begin{aligned} A_i^T P + P A_i < 0 \qquad & (i = 1, \ldots, 2^q) \\ P > 0 \end{aligned}$$

Where $>$ and $<$ indicate positive definite and negative definite constraints respectively. This method was first applied to constant power loads in [7].

III. PROBLEM SETUP

We consider three pulsed load models of increasing complexity. While some pulsed loads (e.g. radar) have multiple power consumption levels, for simplicity we will focus on the case in which a pulsed load wants to transition from drawing no power ($P = 0$) to drawing power P_{on} instantaneously. This corresponds to switching between two equilibrium points, x_{off} and x_{on}. Let Ω_{off} and Ω_{on} be the ROAs of the equilibrium points corresponding to the load being off and on respectively. It is straight-forward to see we can only pulse the load (i.e. transition from x_{off} to x_{on}) if $x_{off} \in \Omega_{on}$. A similar remark holds for transitioning from on to off. However, the models considered here are linear and stable when the load is off, thus $\Omega_{off} = \mathbb{R}^n$ and transitioning from on to off is always stable. Thus we focus solely on the pulsing on case.

For each model we attempt to find the maximum power level P_{max} whose ROA estimate encloses x_{off}, proving we can stably pulse the load. We do this using both the polytopic approach and SOS approach. For the SOS approach, we use a shape factor $h(x)$ that expands the ROA estimate in the direction of x_{off}, as this is the point we hope to enclose.

A. 2nd-Order Pulsed Constant Power Load Model

We first consider a 2nd-order pulsed load model consisting of an inductor-capacitor (LC) filter supplying a constant power load with power P. The continuous dynamics are given by:

$$L\frac{di_L}{dt} = v_{dc} - r_L i_L - v_C$$
$$C\frac{dv_C}{dt} = i_L - \frac{P}{v_C}$$

The parameters are taken from [7] and listed in Table I. As the author's of [7] point out, the large inductance is unrealistic but was chosen to yield a model with a limited region-of-attraction. Let (i_L^{eq}, v_C^{eq}) denote the equilibrium point for a given power P. We shift the equilibrium point to the origin by rewriting the system dynamics in terms of states $x_1 = i_L - i_L^{eq}, x_2 = v_C - v_C^{eq}$.

$$\frac{dx_1}{dt} = \frac{-r_L}{L}x_1 - \frac{1}{L}x_2$$
$$\frac{dx_2}{dt} = \frac{1}{C}x_1 - \left(\frac{1}{v_C^{eq} + x_2}\right)\frac{P}{Cv_C^{eq}}x_2$$

Remark. The term $(v_C^{eq} + x_2)^{-1}$ is the sole source of nonlinearity in constant power load models. This term is not polynomial and therefore we cannot directly apply SOS-based methods. To address this we rewrite the system dynamics as:

$$f(x) = f_0(x) + \frac{g(x)}{h(x)}$$

where

$$f_0(x) = \begin{bmatrix} \frac{-r_L}{L}x_1 - \frac{1}{L}x_2 \\ \frac{1}{C}x_1 \end{bmatrix}, g(x) = \begin{bmatrix} 0 \\ -\frac{P}{Cv_C^{eq}}x_2 \end{bmatrix}$$
$$h(x) = v_C^{eq} + x_2$$

We then multiply condition $-(\nabla V(x) f(x) - l_2(x)) \in \sum[x]$ by $h(x)^2$ to obtain the equivalent condition $-(\nabla V(x)(f_0(x)h(x)^2 + g(x)h(x)) - l_2(x)h(x)^2) \in \sum[x]$. Within the V-s iteration this yields a modified constraint

$$-(\nabla V(x)(f_0(x)h(x)^2 + g(x)h(x)) - l_2(x)h(x)^2) - s_1(x)(V(x) - \gamma) \in \sum[x]$$

We note that a similar approach is done in [9]. Alternatively, one could approximate the term $h(x)$ using a Taylor series expansion.

Using the SOS method, we are able to establish stable pulsed load operation up to 482W. The polytopic method gave identical results. Figure 1 shows the ROA estimate with the pulsed trajectory plotted. Because x_{off} is within our estimate of the Ω_{on} we conclude that the system can pulse to 482W from no load. The trajectory starts at $x_{off} = (0A, 200V)$ and spirals to $x_{on} = (2.44A, 197.3V)$. Figure 2 plots the trajectory against time. The Lyapunov function $V(x)$ is also plotted, confirming its monotonic decrease along the trajectory.

Figure 3 shows the source impedance (2nd-order LC filter) and load impedance (constant power load) at 482W operation. The gain margin is 0.97dB, well below standard design metrics. For this model, it was determined via simulation that pulsing from no load could be stably done up to 537kW at which point the small-signal stability margin is 0.03dB (!). This gap between the 482W and 537W is indicative of how conservative our estimates of the ROA are. At 542kW the system is small-signal unstable. Thus for this model, an operating point being small-signal stable is a surprisingly good indicator that one can transition to this condition from no load.

TABLE I
2ND-ORDER MODEL PARAMETERS

v_{dc}	L	r_L	C
200	$39.5mH$	1.1Ω	$501\mu F$

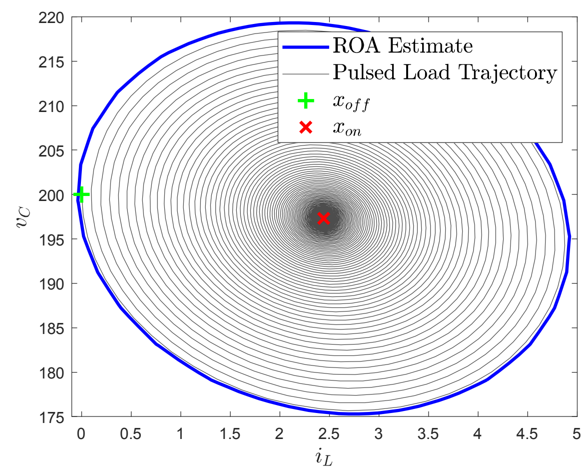

Fig. 1. 2nd-order model ROA estimate for 482W

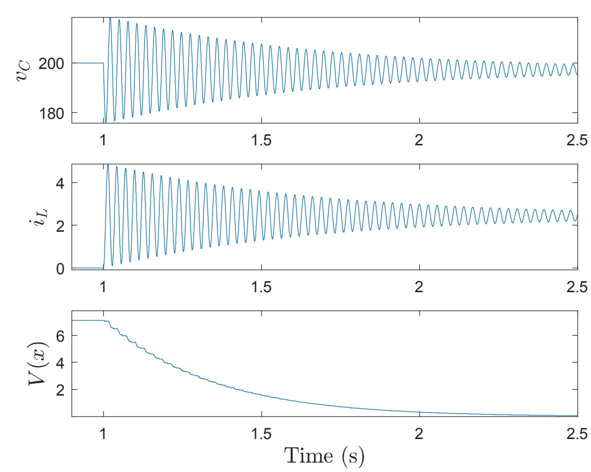

Fig. 2. 2nd-order model pulsing from 0W to 482W

B. 3rd-Order Pulsed Constant Power Load Model

The previous model had unrealistically large inductance and yet still can easily be pulsed from off to on for nearly all points that are small-signal stable. This raises the question of

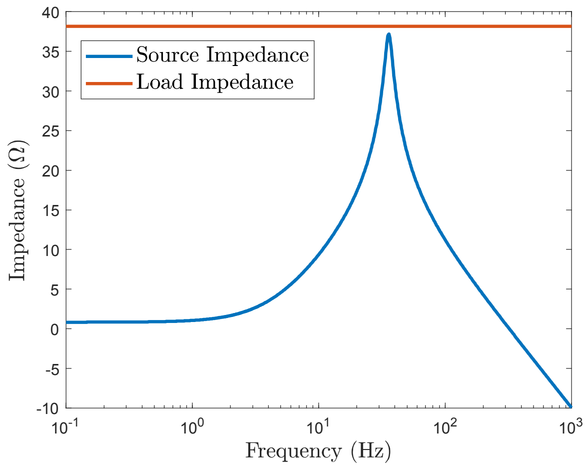

Fig. 3. 2nd-order model source and load impedance

whether large-signal stability is of practical interest for PEDS containing pulsed loads. Restated, can realistic systems have acceptable small-signal stability margins (e.g. 6dB) and yet have unacceptably small regions of attraction?

We switch to a more realistic model in which the inductance and capacitance have reasonable values relative to one another. We then introduce a damping capacitor to minimize the LC resonance. In [15] the following design equation is given for finding the optimal damping resistance r_d to minimize the peak impedance of the LC filter:

$$r_d = R_0 \sqrt{\frac{(2+n)(4+3n)}{2n^2(4+n)}}$$

where $R_0 = \sqrt{\frac{L}{C}}$ and $n = \frac{C_d}{C}$. The capacitance ratio n is typically between 2 to 5. The damping resistance is placed in series with the damping capacitor. The resulting model is:

$$\begin{aligned} L\frac{di_L}{dt} &= v_{dc} - r_L i_L - v_C \\ C\frac{dv_C}{dt} &= i_L - \frac{v_C - v_{Cd}}{r_d} - \frac{P}{v_C} \\ C_d\frac{dv_{Cd}}{dt} &= \frac{v_C - v_{Cd}}{r_d} \end{aligned}$$

We use $n = 4.6$ to achieve 6dB of gain margin when operating at $P_{on} = 1$MW with $V_{dc} = 1000$. Table II lists the parameters. Figure 4 shows the source impedance (3rd-order LC filter) and load impedance (constant power load) at 1MW operation. Despite the larger margin than the 2nd-order model, Figure 5 shows the bus voltage collapse when attempting to pulse to 1MW from a no load condition. If reduced to 997kW (not shown), the load can be pulsed on. Using the SOS method, we are able to establish stable pulsed load operation up to 940kW. This is within 6% of the true maximum capability. The polytopic method yields smaller estimates of the ROA and is only capable of certifying stable pulsed load operation up to 532kW. Figure 6 compares the ROA estimate achieved by both methods for 940kW. Overlaid is the trajectory when pulsing from no load to 940kW. As expected, the trajectory remains within the SOS ellipsoid.

TABLE II

3RD-ORDER MODEL PARAMETERS

v_{dc}	L	r_L	C	C_d	r_d
1000	$4mH$	0Ω	$10mF$	$45mF$	0.3593

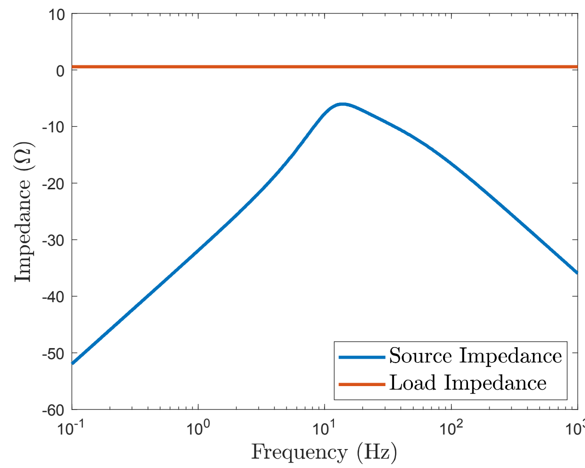

Fig. 4. 3rd-order model source and load impedance

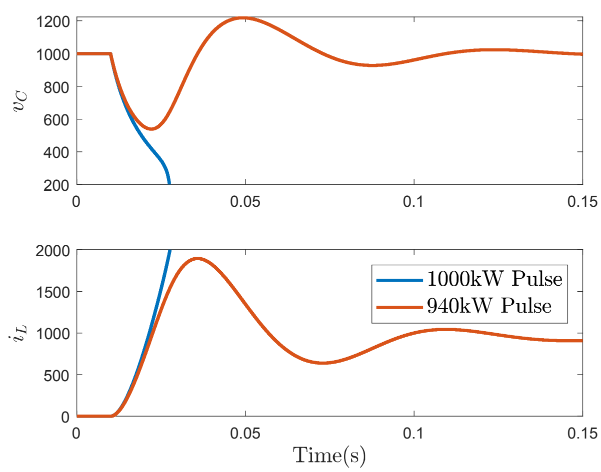

Fig. 5. 3rd-order model pulsing to 940kW and 1000kW (unstable)

C. 5th-Order Pulsed Constant Power Load Model

We add a simple source model to the previous 3rd-order model. The source model consists of bus capacitor C_b with voltage v_{Cb}. A proportional-integral controller tracks the voltage reference v_{ref} for the bus by injecting current into the capacitor. The controller gains are k_p and k_i with associated error state e_v. The injected source current is $i_s = k_p(v_{ref} - v_{Cb}) + k_i e_v$. By appropriate choice of the gains, this model can roughly approximate the voltage regulation

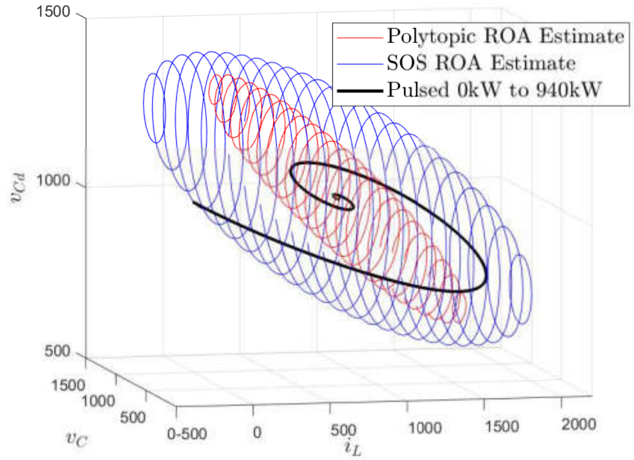

Fig. 6. 3rd-order model ROA Estimate for 940kW

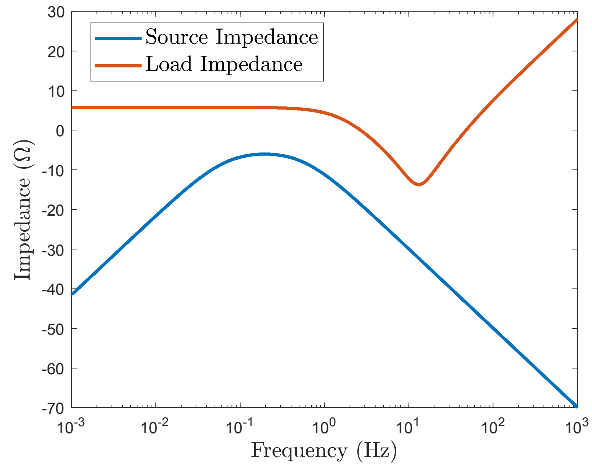

Fig. 7. 5th-order model source and load impedance

dynamics of a power electronic source (high bandwidth) or generator-rectifier unit (low bandwidth). The resulting model is as follows:

$$
\begin{aligned}
C_b \frac{dv_{Cb}}{dt} &= k_p(v_{ref} - v_{Cb}) + k_i e_v - i_L \\
\frac{e_v}{dt} &= v_{ref} - v_{Cb} \\
L\frac{di_L}{dt} &= v_{dc} - r_L i_L - v_C \\
C\frac{dv_C}{dt} &= i_L - \frac{v_C - v_{Cd}}{r_d} - \frac{P}{v_C} \\
C_d \frac{dv_{Cd}}{dt} &= \frac{v_C - v_{Cd}}{r_d}
\end{aligned}
$$

Representative of a large generator-rectifier unit, the voltage regulation loop is tuned to a bandwidth of approximately 0.75Hz. Table III lists the source model parameters. The source impedance and load impedance at the bus interface (v_{Cb}, i_L) is shown in Figure 7. The gain margin of the impedance ratio is 11.7dB.

The previous study established that the load can be stably pulsed up to 997kW when supplied by an ideal voltage source (no impedance). SOS optimization was able to certify stability up to 940kW. With the regulated (non-ideal) source, simulation studies showed that the load is able to be pulsed up to 660kW. The SOS method certifies stable pulsed load operation up at 515kW. The polytopic method is able to certify stable operation up to 46kW. Here we see that the polytopic approach can be extremely conservative.

Figure 8 shows the voltage and current at the bus interface during a 515kW pulse from no load. The 5th-order model has a fast oscillatory mode which quickly decays followed by a slow oscillatory mode. Although we cannot visualize the 5th-order ROA estimates, Figure 9 shows a slice in which the capacitor voltages are set equal and source and load current are set equal. This is representative of equilibrium conditions. The bus voltage v_{Cb} and inductor current i_L in pulsing from no load $(0A, 1000V)$ to 515kW $(515A, 1000V)$ is overlaid.

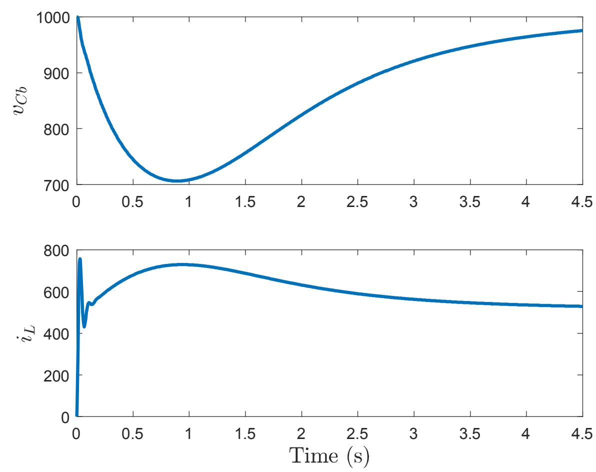

Fig. 8. 5th-order model pulsing from 0kW to 515kW

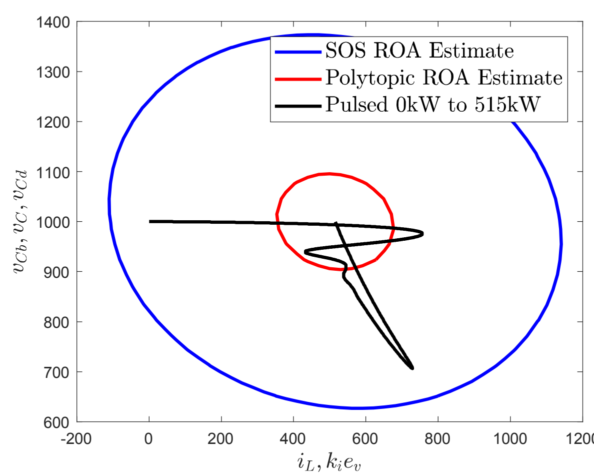

Fig. 9. 5th-order model ROA estimate for 515kW

TABLE III
5TH-ORDER MODEL PARAMETERS

v_{ref}	C_b	k_p	k_i
1000	$500mF$	2	0.75

D. Implementation Details

In all the above examples, we limited our SOS optimization problems to searching over quadratic functions. In future work we plan to explore higher-order polynomials (e.g. quartics) to further reduce the conservatism in our ROA estimates. All examples were solved using MOSEK [16] in conjunction with YALMIP [17] in MATLAB 2017b.

IV. CONCLUSION

In this work we demonstrated how sum-of-squares optimization can be leveraged to analyze the stability of pulsed power loads. Unlike the polytopic approach, the SOS results were not excessively conservative. Further, they provide rigorous proofs of stability in pulsed systems where relying on standard small-signal metrics can be misleading. This suggests that SOS optimization is a useful building block for nonlinear analysis and synthesis tools in naval power systems. In future work we plan to examine more complicated models and leverage SOS optimization methods for synthesizing control algorithms to stabilize pulsed power systems.

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